Davao del Sur State College
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CALCULUS FOR MAED Mathematics Teaching

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PREFACE

Without a secure grounding in first principles, mathematical thinking — like an untethered boat — drifts without direction. This coursepack in Advanced Calculus exists to provide that anchor. Developed for graduate students of the Master of Arts in Education (MAED) at Davao Del Sur State College and undergraduate students of the Bachelor of Science in Mathematics at Surigao Del Sur State University, it offers a rigorous and contextually grounded treatment of Real Analysis across three modules: the real number system, limits and convergence of sequences, and the continuity of functions.

Module 1 establishes the axiomatic foundation — from the Peano Axioms and Mathematical Induction through the field and order properties of the real numbers, the irrationality of surds such as √2 and √5, and a careful treatment of finite, countable, and uncountable sets. Module 2 develops the formal ε–N definition of sequence convergence, proves the Limit Theorems for sequences (scalar, sum, product, and quotient), and extends the limit concept to sequences that oscillate or diverge through the notions of limit superior and limit inferior. Module 3 completes the arc by treating the continuity of functions: the ε–δ definition, the Sequential and Discontinuity Criteria, arithmetic combinations of continuous functions, and the Composition Theorem — establishing that every polynomial is continuous on $\mathbb{R}$ and every rational function is continuous wherever its denominator is nonzero.

Every module follows the Activity–Abstraction–Application (AAA) framework: intuitive exploration precedes formal theory, and formal theory is always connected back to practical interpretation. Scenarios drawn from Davao del Sur — durian harvests in Digos City, tilapia production at Lake Sebu, jeepney fare structures, tidal patterns in Sarangani Bay, and barangay census data — ground abstract concepts in the world the learner already knows. These are not ornamental; they model the kind of contextualised mathematics instruction that graduate educators are themselves expected to deliver.

At the same time, these materials make no concession on rigor. Every definition is stated precisely, every theorem is proved in full, and students are expected to cite theorems at each step, to distinguish intuition from proof, and to supply rigorous arguments wherever informal reasoning falls short. Assessments are graduated accordingly: from exercises building

definitional fluency, through proof construction problems, to advanced applications requiring original arguments set in regional contexts.

It is the author's hope that students who work through these modules will emerge not only with a stronger command of real analysis, but with the confidence to pass rigorous mathematical thinking on to others. The real numbers, convergent sequences, and continuous functions are the foundations beneath every calculation and every model we apply to the world. To understand them is to stand on ground that will not shift.

May this coursepack serve as a reliable anchor for the mathematical journeys ahead.

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