Davao del Sur State College
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MODERN GEOMETRY: Euclidean and Non-Euclidean

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PREFACE

This module, *Lesson Module in Modern Geometry (MATH 114)*, is designed for students of the Bachelor of Science in Mathematics program at Davao del Sur State College. It guides learners through four interconnected domains — plane Euclidean geometry, affine transformations, spherical geometry, and finite axiomatic systems — with the conviction that mathematical rigor and genuine understanding are inseparable. Every theorem presented here is proved in full, and every concept is developed with the precision that graduate-preparatory mathematics demands.

Each lesson follows the 4-A's instructional framework — Activity, Analysis, Abstraction, Application — anchored to CMO No. 48, s. 2017. This sequence reflects a fundamental pedagogical principle: mathematical understanding is constructed, not simply transmitted. Students first engage with a concrete, culturally grounded task; they examine it critically; they arrive at formal definitions and theorems as informed participants rather than passive recipients; and they apply what they have learned to real contexts. The Activity stage is not a warm-up to be hurried past — it is the epistemological foundation on which the entire lesson rests.

The contexts drawn from Davao del Sur and the wider Philippine experience — displacement vectors traced along jeepney routes from Digos to Sta. Cruz, perpendicular distances from city halls to irrigation canals, great-circle flight paths from Davao to Tokyo, the dihedral symmetry of T'boli t'nalak weaving and the eight-rayed sun of the Philippine flag, Fano-plane tournament schedules for Digos City barangay basketball leagues, and the Hamming error-correcting codes underlying Mindanao's fiber-optic networks — are not decorative. They demonstrate that abstract mathematics is not confined to textbooks. They are chosen to show, concretely and repeatedly, that the structures studied in this module are the very structures that organize space, navigation, culture, and communication in the world students already inhabit.

Module 1 builds the Euclidean plane from the vector space $\mathbb{R}^2$, establishing the inner product, norm, distance, and the full classification of plane isometries, culminating in the dihedral symmetry groups of the Euclidean group E(2). Module 2 extends to affine geometry, where the Fundamental Theorem of Affine Geometry, shear transformations, dilatations, and similarities are developed through the lens of oblique aerial photography and national cartographic practice. Module 3 moves to the sphere S^2, introducing great circles as geodesics, the spherical triangle inequality, and Euler's Rotation Theorem — which reveals that every non-trivial rotation of the globe fixes exactly two antipodal points. Module 4 enters finite geometry, constructing three-point, four-point, and Fano geometries from axioms, proving the projective plane counting formula $n^2 + n + 1$, and connecting the Fano Plane to error-correcting codes in modern telecommunications.

The module is addressed primarily to third- and fourth-year BS Mathematics students. A working familiarity with linear algebra and introductory proof-writing is assumed. Students who encounter gaps in prerequisite knowledge will find that the Activity stages are deliberately designed to reconstruct essential ideas through exploration before formal

treatment begins. Readers are urged above all to engage with the proofs — not as obstacles, but as the clearest expression of what it means to do mathematics rather than merely learn about it.

Geometry, in all its forms, is a study of what is preserved and what is transformed as we move through space, shift our perspective, and build new worlds from first principles. It is the sincere hope of the authors that this module preserves something essential in every student who works through it: the discipline of careful reasoning, the satisfaction of a completed proof, and the enduring conviction that mathematics, at its best, illuminates the world we actually live in.

May this coursepack serve as a reliable anchor for the mathematical journeys ahead.

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